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SHORT COMMUNICATION

RADIATION EFFECTS ON MHD FREE CONVECTION FLOW OF A GAS PAST A SEMI-INFINITE VERTICAL PLATE

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ABSTRACT

Free convection heat transfer due to the simultaneous action of buoyancy, radiation and transverse magnetic field is investigated for a semi-infinite vertical plate. Solutions are derived by expanding the stream function and the temperature into a series in terms of the parameter $\xi = x^{1/2}L^{-1/2}$, where L is the length of the plate. Velocity and temperature functions are shown on graphs and the numerical values of functions affecting the shear stress and the rate of heat transfer are entered in a table. The effects of the magnetic field parameter λ and the radiation parameter F on these functions are discussed.

KEY WORDS MHD Radiation Free convection

NOMENCLATURE

B_0	magnetic field
C_p	specific heat
$e_{b\lambda}$	Plank's function
f	dimensionless stream function
F	radiation parameter
g	acceleration due to gravity
Gr	Grashof number
h	heat transfer coefficient
k	thermal conductivity
K_λ	absorption coefficient
L	length of the plate
Pr	Prandtl number
q_r	radiative heat flux
T	temperature
u, v	velocity components in the x and y directions, respectively
x	streamwise coordinate
y	direction normal to the plate
α	thermal diffusivity
β	coefficient of volume expansion
λ	magnetic field parameter
η, ξ	pseudosimilarity variables
θ	dimensionless temperature
μ	coefficient of viscosity
ρ	density
σ	scalar electrical conductivity of the fluid
ψ	stream function
τ	shear stress
Subscripts	
w	properties of the wall
∞	free stream conditions

INTRODUCTION

Free convection flows past different types of vertical bodies are studied because of their wide application. Free convection flow of pure fluids past a semi-infinite vertical plate, at normal temperature, was first presented by Pohlhausen[6] who effected a solution by the momentum

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integral method. Ostrach[5] solved this problem, for the first time, using a similarity method. The fluid considered was air. Later on, this problem of free convection past a semi-infinite vertical plate was studied extensively under different physical conditions by many authors. These have been referred to in a recently published reference book by Gebhart *et al.*[2]. In all these studies, the effects of radiation are not studied extensively. In space technology applications and at higher operating temperatures, radiation effects can be quite significant. Since radiation is quite complicated, many aspects of its effect on free convection or combined convection have not been studied in recent years. However, Greif *et al.*[3] have shown that in the optically thin limit, the physical situation can be simplified and, thereby, they derived an exact solution to the problem of fully developed radiating laminar convective flow in an infinite vertical heated channel. Greif *et al.*[3] followed closely the analysis of Cogley *et al.*[1] who showed that, for an optically thin limit, the fluid does not absorb its own emitted radiation, i.e., there is no self-absorption, but the fluid does absorb radiation emitted by the boundaries. Cogley *et al.*[1] showed that, in the optically thin limit for a gray-gas near equilibrium, the following relation holds:

$$\frac{\partial q_r}{\partial y} = 4(T - T_w)I \tag{1}$$

where

$$I = \int_0^\infty K_{\lambda w} \left(\frac{\partial e_{b\lambda}}{\partial T} \right)_w \mathrm{d}\lambda$$

All the physical quantities are defined in the nomenclature. Further simplification can be made concerning the spectral properties of radiating gases (Tien[7]), but these are not necessary for the present analysis. In the optically thin limit for gray-gas, this is the first analysis of free convection flow.

In space technology and in nuclear engineering applications, such a problem is quite common. But in these fields, the presence of a magnetic field plays an important role and these effects have not been studied in the case of free convective flow of a radiating gas under the above mentioned conditions. Hence, we propose investigating the situation where buoyancy, radiation and magnetic field act simultaneously.

MATHEMATICAL CONCEPTS

Here, the flow of an electrically conducting, gray gas near equilibrium in the optically thin limit past a semi-infinite vertical plate is assumed in the x-direction which is taken along the plate in the vertically upward direction. The y-axis is assumed to be normal to the plate. Also, a magnetic field of constant intensity is assumed to be applied normal to the vertical plate, *Figure 1*. It is also assumed, a valid assumption on laboratory scale, that the induced magnetic field is

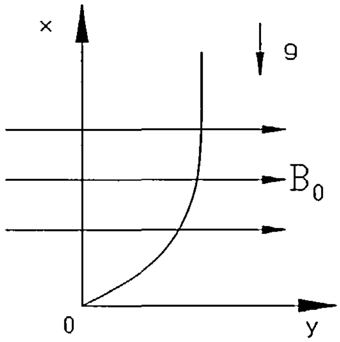

Figure 1 Schematic diagram

negligible. Since the velocity of the fluid is low, the viscous dissipative heat is assumed to be negligible. Then the steady free convection flow is governed by the following equations,

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{2}$$

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=\mu\frac{\partial^2 u}{\partial y^2}+\rho g\beta(T-T_\infty)-\sigma B_0^2 u \tag{3}$$

$$\rho C_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k\frac{\partial^2 T}{\partial y^2}-\frac{\partial q_r}{\partial y} \tag{4}$$

All the physical variables are defined in the nomenclature. The boundary conditions are given by,

$$\begin{aligned} &u=0, \quad v=0, \quad T=T_w \quad \text{at } y=0\\ &u=0, \quad T\to T_\infty \quad \text{as } y\to\infty \end{aligned} \tag{5}$$

These are the usual no-slip boundary conditions at the plate which is also assumed to be at constant temperature T_w.

As similarity solutions are not possible in the present case, due to the presence of the radiation term, we seek a series solution. The series solution method has been used by several investigators who have found that this approach yields accurate results. Proceeding with the analysis, we now define the similarity variables as follows,

$$\begin{aligned} &C=\left(\frac{g\beta\,\Delta T}{4\nu^2}\right)^{1/4}=\left(\frac{Gr}{4}\right)^{1/4}L^{-3/4}, \qquad \Delta T=T_w-T_\infty\\ &\eta=Cyx^{-1/4}, \qquad Gr=\frac{g\beta L^3\,\Delta T}{\nu^2}, \qquad Pr=\nu/\alpha\\ &\lambda=\frac{\sigma B_0^2 L^2}{\rho\nu Gr^{1/2}}, \qquad F=\frac{4IL^2}{\rho C_p\nu Gr^{1/2}}\\ &\psi=4\nu Cx^{3/4}f(\xi,\eta), \qquad \xi=x^{1/2}L^{-1/2} \end{aligned} \tag{6}$$

The continuity equation is now satisfied by

$$u=\frac{\partial\psi}{\partial y}, \qquad v=-\frac{\partial\psi}{\partial x} \tag{7}$$

where ψ is a stream-function.

From (6) and (7), we can show that,

$$\begin{aligned} &u=4\nu C^2x^{1/2}f'(\xi,\eta)\\ &v=-\nu Cx^{-1/4}\left(3f+2\xi\frac{\partial f}{\partial\xi}-\eta f'\right) \end{aligned} \tag{8}$$

The primes above indicate differentiation with respect to η only. In view of (6), (7) and (8), we can show that (2)–(4) and (5) reduce to,

$$f'''+3ff''-2f'^2+2\xi\left(f''\frac{\partial f}{\partial\xi}-f'\frac{\partial^2 f}{\partial\xi\,\partial\eta}\right)+\theta-2\xi\lambda f'=0 \tag{9}$$

$$\frac{1}{Pr}\theta''+3f\theta'+2\xi\left(\theta'\frac{\partial f}{\partial\xi}-f'\frac{\partial\theta}{\partial\xi}\right)-2\xi F\theta=0 \tag{10}$$

These equations are still not ordinary differential equations and to reduce these to ordinary differential equations, we expand f, θ in powers of ξ as follows,

$$f(\xi,\eta)=f_0(\eta)+\xi f_1(\eta)+\xi^2 f_2(\eta)+\cdots$$
$$\theta(\xi,\eta)=\theta_0(\eta)+\xi\theta_1(\eta)+\xi^2\theta_2(\eta)+\cdots \tag{11}$$

The above two series expansions are known to be convergent for $\xi\leqslant 1$. For $\xi>1$, one can use Shanks[8] transformations to make the series convergent. This was not found necessary in the present study since we have considered $\xi<1$.

Substituting (11) into (9) and (10), equating coefficients of equal powers of ξ, neglecting those of ξ^3, we have the following set of ordinary differential equations,

$$f_0'''-2f_0''+3f_0f_0''+\theta_0=0 \tag{12}$$
$$f_1'''-4f_0'f_1'+3(f_0f_1''+f_1f_0'')+2(f_1f_0''-f_0'f_1')+\theta_0-2\lambda f_0'=0 \tag{13}$$
$$f_2'''-2((f_1')^2+2f_0f_2)+3(f_0f_2''+f_1f_1''+f_2f_0'')+2[(f_1f_1''+2f_2f_0''-(2f_0'f_2'+(f_1')^2)]+\theta_2-2\lambda f_1'=0 \tag{14}$$
$$\theta_0''+3Prf_0\theta_0'=0 \tag{15}$$
$$\theta_1''+3Pr(f_0\theta_1'+f_1\theta_0'+2Pr(f_1\theta_0'-f_0'\theta_1)-2PrF\theta_0=0 \tag{16}$$
$$\theta_2''+3Pr(f_0\theta_2'+f_1\theta_0'+2Pr[(f_1\theta_1'+2f_2\theta_0')-(f_1'\theta_1+2f_0'\theta_2)]-2PrF\theta_1=0 \tag{17}$$

Here the primes represent differentiation with respect to η. The boundary conditions (5) in view of (8) and (11) now reduce to,

$$f_0(0)=0,\quad f_0'(0)=0,\quad f_1(0)=f_1'(0)=0,\quad f_2(0)=f_2'=0$$
$$\theta_0(0)=1,\quad \theta_1(0)=\theta_2(0)=0 \tag{18}$$
$$f_1(\infty)=f_1'(\infty)=f_2'(\infty)=0$$
$$\theta_0(\infty)=\theta_1(\infty)=\theta_2(\infty)=0$$

Equations (12)–(17), subject to boundary conditions (18) are solved on a high speed computer using 2-point boundary value shooting techniques and the functions f_1', f_2', θ_1 and θ_2 are shown in *Figures 2–5*. The functions f_0' and θ_0 are already well-known.

Numerical procedure

The numerical procedure used, solves the two-point boundary value problem for a system of N ordinary differential equations, in the range (x_0, x_1) of the form,

$$\frac{dy_i}{dx}=f_i(x, y_1, y_2, \ldots, y_N),\qquad i=1,2,\ldots,N$$

and the derivatives f_i are calculated by a procedure that evaluates the derivatives of y_i at any point x. Initially, N boundary values of the variables y_i must be specified some at x_0 and some at x_1. We then supply estimates of the remaining N boundary values and the procedure corrects these by a form of Newton iteration. Starting from the known and estimated values of y_i at x_0 the procedure integrates the equations forward to a matching point R using Merson's method; similarly starting from x_0, it integrates backwards to R. The difference between the forward and backward values of y_i at R should be zero for a true solution. The procedure uses a generalised Newton method to reduce these differences to zero by calculating corrections to the estimated boundary values. This process is repeated iteratively until convergence is obtained to a given level of accuracy.

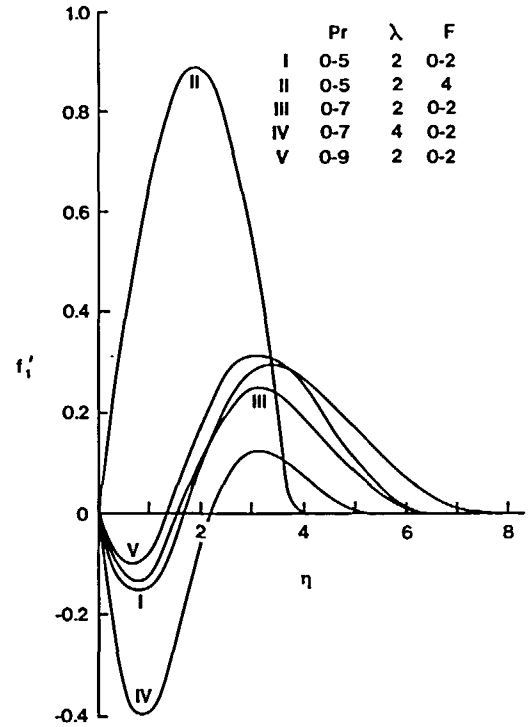

Figure 2 Distribution of velocity function f'_1

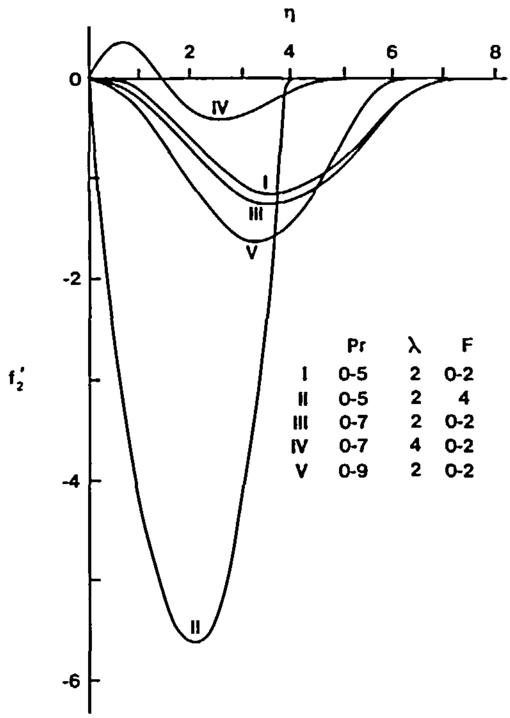

Figure 3 Distribution of velocity function f'_2

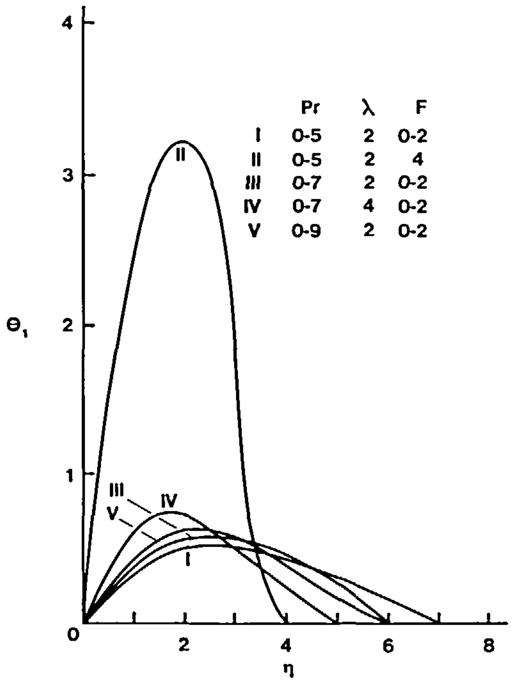

Figure 4 Distribution of temperature function θ_1

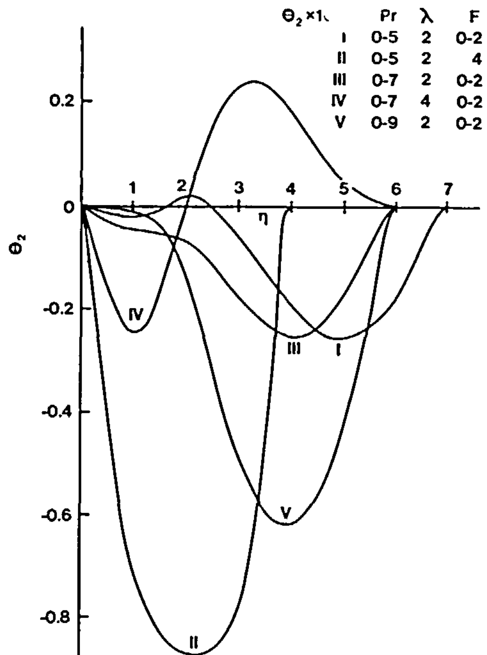

Figure 5 Distribution of temperature function θ_2

Table 1 Values of wall shear stress and heat transfer rate functions

		$f''_0(0)$		$-f''_1(0)$			$f''_2(0)$	
Pr	λ/F	0	0	0.2	4	0	0.2	4
0.9	2	0.7126	0.4322	0.4731	1.2493	0.2937	0.2906	1.2918
0.733	2	0.6784	0.4566	0.4955	1.2309	0.3000	0.2982	1.2431
	4			0.9512			1.1882	
0.5	2	0.6530	0.5042	0.5395	1.2053	0.3160	0.3166	1.1723
	4		1.0077	1.0428		1.2592	1.2592	
		$\theta'_1(0)$		$\theta'_1(0)$			$-\theta'_2(0)$	
0.9	2	0.5465	0.2650	0.0498	−4.0392	0.0918	0.1530	3.2328
0.733	2	0.5079	0.2490	0.0590	−3.5510	0.0775	0.1325	2.5982
	4			0.3048			0.4330	
0.5	2	0.4412	0.2205	0.0690	−2.8090	0.0570	0.1018	−2.7216
	4		0.4379	0.2865		0.2311	0.3286	

From the velocity field, we can now study the skin friction. It is given by,

$$\tau = -\mu \left(\frac{\partial u}{\partial y} \right)_{y=0} \tag{19}$$

and in view of (6) and (8), it reduces to,

$$\tau = -4\mu\nu C^3 x^{1/2} f''(\xi, 0) \tag{20}$$

The numerical values of $f''_0(0)$, $f''_1(0)$ and $f''_2(0)$ are presented in *Table 1*.

The heat transfer coefficient is given by,

$$h = -k\Delta T C x^{-1/4} \theta'(\xi, 0) \tag{21}$$

We have calculated the numerical values of $\{-\theta'_0(0)\}$, $\{\theta'_1(0)\}$ and $\{-\theta'_2(0)\}$ and these are presented in *Table 1*.

DISCUSSION

It is observed here that radiation does affect the velocity and temperature field of free convection flow of an electrically conducting fluid past a semi-infinite vertical plate in the presence of a transverse magnetic field.

Figure 2 shows a rise in the value of the velocity function f'_1 for higher values of the radiation parameter F when the Prandtl number and the magnetic field parameter are constant. This effect is more pronounced for lower values of the Prandtl number. *Figure 3* shows that the fall of the velocity function f'_2 below its initial value of zero is higher for higher values of the radiation parameter F when the magnetic field parameter λ and Pr are fixed. However, for high Prandtl number gases, radiation effects are more pronounced. *Figure 4* shows the development of the temperature function θ_1 which increases with increasing values of F when Pr and λ are held constant. When the Prandtl number of gases decreases, radiation effects are more pronounced. *Figure 5* shows the fall of the temperature function θ_2, below zero and θ_2 decreases more with increasing the radiation parameter F, at constant values of λ and Pr.

From *Table 1*, we conclude that the function $\{-f''_1(0)\}$ increases with increasing the radiation parameter F at fixed values of λ and Pr but the function $f''_2(0)$ decreases initially at small values of the radiation parameter F for gases with large Pr; but for gases with small Pr and λ, $f''_2(0)$ is found to decrease owing to the presence of radiation.

In the presence of radiation, a decrease in the Prandtl number leads to a rise in the value of $\theta'_1(0)$ when the magnetic field parameter λ is constant, but $\{-\theta'_2(0)\}$ decreases with decreasing the Prandtl number. Again $\theta'_1(0)$ decreases and $\theta'_2(0)$ increases with increasing the radiation parameter F when Pr and λ are held constant.

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